CS/MATH111 ASSIGNMENT 3

Problem 1: Let W_n be the number of strings of length n formed from letters A, B, C, and D that do not contain a substring AA, BA or CA. For example, for $n = 2$, all the strings with this property are

$$AB, AC, AD, BB, BC, BD, CB, CC, CD, DA, DB, DC, DD$$

and thus $W_2 = 13$. (Note that $W_0 = 1$, because the empty string satisfies the condition.)
(a) Derive a recurrence relation for the numbers W_n. Justify it.
(b) Find the formula for the numbers W_n by solving this recurrence. Show your work.

Problem 2: Solve the following recurrence equation:

$$\begin{aligned} f_n &= 13f_{n-2} + 12f_{n-3} + 2n + 1 \\ f_0 &= 0 \\ f_1 &= 1 \\ f_2 &= 1 \end{aligned}$$

Show your work (all steps: the associated homogeneous equation, the characteristic polynomial and its roots, the general solution of the homogeneous equation, computing a particular solution, the general solution of the non-homogeneous equation, using the initial conditions to compute the final solution.)

Problem 3: Solve the following recurrence equation:

$$\begin{aligned} t_n &= t_{n-1} + 2t_{n-2} + 3^n \\ t_0 &= 0 \\ t_1 &= 4 \end{aligned}$$

Show your work (all steps: the associated homogeneous equation, the characteristic polynomial and its roots, the general solution of the homogeneous equation, computing a particular solution, the general solution of the non-homogeneous equation, using the initial conditions to compute the final solution.)